BME 315/253 **HOMEWORK #1 Solution** **Spring 08**

1.

(a)

$$J_{Ca} = J_{Drift} + J_{Diffusion}$$
$$= -\mu z \frac{dv}{dx}\left[Ca^{+2}\right] - \frac{KT}{q}\mu_{Ca}\frac{d\left[Ca^{+2}\right]}{dx}$$
$$= -2\mu_{Ca}\left[Ca^{+2}\right]\frac{dv}{dx} - \frac{KT\mu_{Ca}}{q}\frac{d\left[Ca^{+2}\right]}{dx}$$

(b)

$J_{Ca} = 0$ at equilibrium, therefore,

$$0 = -2\mu_{Ca}\left[Ca^{+2}\right]\frac{dv}{dx} - \frac{KT\mu_{Ca}}{q}\frac{d\left[Ca^{+2}\right]}{dx}$$

$$2\left[Ca^{+2}\right]\frac{dv}{dx} = \frac{-KT}{q}\frac{d\left[Ca^{+2}\right]}{dx}$$

$$\int_{Vi}^{V_o} dv = \frac{-KT}{2q}\int_{\left[Ca^{+2}\right]_i}^{\left[Ca^{+2}\right]_o}\frac{d\left[Ca^{+2}\right]}{\left[Ca^{+2}\right]}$$

$$V_o - V_i = \frac{-KT}{2q}\ln\frac{\left[Ca^{+2}\right]_o}{\left[Ca^{+2}\right]_i}$$

$$E_{Ca} = V_i - V_o = \frac{KT}{2q}\ln\frac{\left[Ca^{+2}\right]_o}{\left[Ca^{+2}\right]_i}$$

2.

For sodium:

$$J_{Na} = -\mu_{Na} z_{Na}\frac{dv}{dx}\left[Na^{+}\right] - \frac{KT}{q}\mu_{Na}\frac{d\left[Na^{+}\right]}{dx}$$

Assume

$\frac{dv}{dx} = \frac{V}{\delta}$, thus

$$J_{Na} = -\frac{P_{Na}}{q} V\left[Na^+\right] - P_{Na}\,\delta \frac{d\left[Na^+\right]}{dx}$$

with $P_{Na} = \frac{\mu_{Na^+} KT}{\delta q}$. Rearranging and integrating yields:

$$\int_o^{\delta} dx = -\int_{\left[Na^+\right]_i}^{\left[Na^+\right]_o} \frac{d\left[Na^+\right]}{\frac{J_{Na}}{P_{Na}\delta} + \frac{qV\left[Na^+\right]}{KT\delta}}$$

and

$$1 = -\frac{KT}{qV} \ln\left(\frac{\frac{J_{Na}}{P_{Na}\delta} + \frac{qV\left[Na^+\right]_o}{KT\delta}}{\frac{J_{Na}}{P_{Na}\delta} + \frac{qV\left[Na^+\right]_i}{KT\delta}}\right)$$

Rearranging gives:

$$e^{-\frac{qV}{KT}} = \frac{\frac{J_{Na}}{P_{Na}\delta} + \frac{qV\left[Na^+\right]_o}{KT\delta}}{\frac{J_{Na}}{P_{Na}\delta} + \frac{qV\left[Na^+\right]_i}{KT\delta}}$$

Solving for J_{Na} gives:

$$J_{Na} = \frac{qVP_{Na}}{KT}\left(\frac{\left[Na^+\right]_o - \left[Na^+\right]_i e^{-\frac{qV}{KT}}}{e^{-\frac{qV}{KT}} - 1}\right)$$

With space charge neutrality and equation 3.27 and 3.28

$$J_{Na} + J_K = J_{Cl}$$

$$P_{Na}\left(\left[Na^+\right]_o-\left[Na^+\right]_i e^{-\frac{qV}{KT}}\right)+P_K\left(\left[K^+\right]_o-\left[K^+\right]_i e^{-\frac{qV}{KT}}\right)$$

$$=P_{Cl}\left(\left[Cl^-\right]_o-e^{-\frac{qV}{KT}}-\left[Cl^-\right]_i\right)$$

Solving for V gives:

$$e^{-\frac{qV}{KT}}=\frac{P_{Na}\left[Na^+\right]_o+P_K\left[K^+\right]_o+P_{Cl}\left[Cl^-\right]_i}{P_{Na}\left[Na^+\right]_i+P_K\left[K^+\right]_i+P_{Cl}\left[Cl^-\right]_o}$$

or

$$V=V_o-V_i=\frac{-KT}{q}\ln\left(\frac{P_{Na}\left[Na^+\right]_o+P_K\left[K^+\right]_o+P_{Cl}\left[Cl^-\right]_i}{P_{Na}\left[Na^+\right]_i+P_K\left[K^+\right]_i+P_{Cl}\left[Cl^-\right]_o}\right)$$

3.

(a). $$J_{Cl}=-\mu_{Cl}\frac{KT}{q}\frac{d[Cl^-]}{dx}-\mu_{Cl}z_{Cl}[Cl^-]\frac{dv}{dx}$$

$$J_{Cl}=-\mu_{Cl}\frac{KT}{q}\quad\frac{d[Cl^-]}{dx}+\mu_{Cl}[Cl^-]\frac{dv}{dx}$$

$$J_B=-\mu_B\frac{KT}{q}\frac{d[B^{+3}]}{dx}-\mu_B z_B[B^{+3}]\frac{dv}{dx}$$

$$=-\mu_B\frac{KT}{q}\frac{d[B^{+3}]}{dx}-3\mu_B[B^{+3}]\frac{dv}{dx}$$

At equilibrium $J_{Cl}=J_B=0$. Thus, the Nernst potential are:

$$E_{Cl}=\frac{-KT}{q}\ln\frac{[Cl^-]_o}{[Cl^-]_i}$$

$$E_B=\frac{KT}{3q}\ln\frac{[B^{+3}]_o}{[B^{+3}]_i}$$

$$E_B=E_{Cl}$$

$$\ln\frac{[Cl^-]_i}{[Cl^-]_o} = \ln\left(\frac{[B^{+3}]_o}{[B^{+3}]_i}\right)^{1/3}$$

or

$$\frac{[Cl^-]_i}{[Cl^-]_o} = \left(\frac{[B^{+3}]_o}{[B^{+3}]_i}\right)^{1/3}$$

(b) Conservation of mass:

$$[Cl^-]_i + [Cl^-]_o = 1,600$$

$$[B^{+3}]_i + [B^{+3}]_o = 500$$

Space Charge Neutrality:

$$[Cl^-]_i = 3[B^{+3}]_i$$

and

$$3[B^{+3}]_o + 100 = [Cl^-]_o$$

Donnan equilibrium:

$$\frac{[Cl^-]_i^3}{[Cl^-]_o^3} = \frac{[B^{+3}]_o}{[B^{+3}]_i}$$

$$\frac{27[B^{+3}]^3{}_i}{\left(1600 - 3[B^{+3}]_i\right)^3} = \frac{500 - [B^{+3}]_i}{[B^{+3}]_i}$$

$$27[B^{+3}]^4{}_o = \left(1600 - 3[B^{+3}]_i\right)^3\left(500 - [B^{+3}]_i\right)$$

$$5.67\text{x}10^4[B^{+3}]^3{}_i - 4.464\text{x}10^7[B^{+3}]^2{}_i + 1.5616\text{x}10^{10}[B^{+3}]_i + 2.048\text{x}10^{12} = 0$$

Solving yields:

$$[B^{+3}]_i = 262.3mM$$

$$[B^{+3}]_o = 500 - [B^{+3}]_i = 237.3mM$$

4. (Just Part (a)).

(a).

$$E = \frac{KT}{zq}\ln\frac{[I]_o}{[I]_i} = \frac{26}{z}\ln\frac{[I]_o}{[I]_i}\,mv$$

$$E_K = 26\ln\frac{6}{168}\,mv = -87mv$$

$$E_{Na} = 26\ln\frac{337}{50}\,mv = 50mv$$

$$E_{Cl} = -26\ln\frac{340}{41}\,mv = -55mv$$